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Optimal taxation, critical-level utilitarianism and economic growth*

Spataro[°] and by Luca Spataro[°] and Thomas I. Renström^{*}

Abstract

In this work we analyze the issue of taxation in an intertemporal economy with endogenous fertility under critical-level utilitarianism, both from a positive and normative standpoint. On the positive side we analyse the effects of a change in the tax on capital income and on the population size, both separately and in a policy aiming at maintaining per-capita debt constant. On the normative side, we characterize the first-best and second-best optimal tax structures both when labour supply is exogenous and endogenous.

Keywords: Taxation, endogenous fertility, critical level **utilitarianism**, population.

J.E.L. Classification: D63, H21, J13, O40.

1. Introduction

The issue of optimal taxation is a long-debated subject in economics. However, only recently the consequences of endogenous fertility have been explored. In fact, traditionally the two topics have been analysed separately: on the one hand, the problem of optimal taxation in dynamic general equilibrium models has been investigated extensively: see Atkinson and Stiglitz (1972) for the earliest results on finite-time economies; Judd (1989) and Chamley (1986) for the results in infinite horizon economies based on Ramsey (1928); Erosa and Gervais (2002) and De Bonis and Spataro (2010) for overlapping-generations economies; and Basu, Marsiliani, and Renström (2004) and Basu and Renström (2007) for indivisible labour economies.

On the other hand, another strand of literature has been focusing on the optimal population growth rate (Samuelson 1975, Deardorff 1976 and, more recently, Jaeger and Kuhle 2009 and Renström and Spataro 2010) and on the role of endogenous fertility on optimal welfare state design (in particular social security, see, for example, Cigno and Rosati 1992, Zhang and Nishimura 1992 and 1993, Cremer, Gahvari and Pestieau 2006, Yew and Zhang 2009, Meier and Wrede 2010).

In this paper we aim at addressing the issue of optimal taxation in presence of endogenous fertility in a unified framework. In particular, we tackle such an issue by assuming that agents are entitled with “critical-level utilitarian preferences” (see Blackorby et al. 1995)¹. Critical-level utilitarianism is an axiomatically founded population principle that can avoid the repugnant conclusion (see Parfit 1976, 1984, Blackorby et al. 1995 and 2002). The latter implies that any state in which each member of the population enjoys a life above “neutrality” is declared inferior to a state in which each member of a larger population lives a life with lower utility. Indeed, such a result is likely to emerge in economic models under classical utilitarianism (CU) and endogenous fertility, that is in

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¹ Among other non utilitarian principles, seem, for example, Golosov, Jones and Tertilt (2007).

presence of social orderings based on the (sum of) well-being (i.e. utilities) of the individuals who are alive in different states of the world.

Indeed, there are several ways for avoiding the repugnant conclusion. Some earlier literature assumed objective functions of a particular form.² However, such objective functions may not have an axiomatic foundation. We believe an axiomatic foundation is important, especially because we are dealing with questions regarding life (who will live and who will not)³. In fact, in a twin paper (Renström and Spataro 2010) we have shown that critical level utilitarianism (CLU) can deliver a steady state equilibrium entailing an interior solution for the rate of growth of population, provided that the critical level belongs to a positive, open interval. We recall here that the critical level α can be defined as the utility level of an extra-individual i who, if added to an unaffected population N with utility distribution u , would make the two alternatives socially indifferent, i.e. (N, u) as good as $(N, u; i, \alpha)$.

In the present work we rely on the work by Blackorby et al. (1997) allowing for the possibility of discounting the utilities of future generations. However, we depart from the existing literature in that we tackle the issue of taxation, both from a positive and normative standpoint, in a general equilibrium setting, with endogenous population and CLU. To the best of our knowledge, this has not been done before⁴.

The paper is organized as follows: after presenting the model, in section 3 we characterise the steady state equilibrium and, in section 4 we perform a comparative statics analysis in order to assess the effect of taxes on the equilibrium levels of consumption and population growth rate. Finally, in section 5 we characterize the optimal structure of taxes both in absence and in presence of endogenous labour supply.

2. The economy

We assume, for the sake of simplicity, that each generation lives for one period, and life-time utility is $u(c_t)$, where c_t is life-time consumption for that individual. This means that generations will not overlap⁵. We also follow the convention that $u = 0$ represents neutrality at individual level (i.e. if $u < 0$ the individual prefers not to have been born), and denote the critical level as α . We start our analysis by assuming that labour supply, l , is exogenously fixed and normalized to 1; we will relax this assumption in section 5.2. An individual family chooses consumption, savings and the number of children (i.e. the change in the cohort size N). As for firms, we assume perfectly competitive markets and constant return to scale technology. The consequence of the assumptions on the production side is that we retain the “standard” second-best framework, in the sense that there are no profits and the competitive equilibrium is Pareto efficient in absence of taxation. Otherwise there would be corrective elements of taxation. Finally, we assume the government finances an exogenous stream of expenditure by issuing debt and levying taxes.

To retain the second-best, we levy taxes on the choices made by the families, i.e. savings and population. Consequently we introduce the capital-income tax and a population tax proportional to the number of children. Furthermore, we allow the government to levy a tax on the choice each generation makes about the size of the next generation. Regarding

² E.g. Barro and Becker (1988) and Becker and Barro (1989).

³ Therefore, although several authors have criticised CLU, such as Parfit 1976 and 1984, Hurka 1983 and 2000, Arrenius 2000, Hg 1986, Shiell 2008 -see Blackorby (2005) and Renström and Spataro (2010) for a discussion of such critiques- we decided to maintain such an approach.

⁴ A similar problem of optimal fertility has been dealt with by Barro and Becker (1988) and Becker and Barro (1989), although they do not deal either with critical level utilitarianism or with taxation.

⁵ This assumption can be relaxed without changing the fundamental properties of the model.

the population tax, it does not matter if we tax the present generation or the future, because of altruism. For simplicity we assume that the children pay the population tax, making it proportional to N and when parents make choice of number of children they take into account this tax liability and resulting reduction in their children's consumption. Consequently the population tax distorts population choice.

2.1. Individuals

The problem of each household is to maximize the following birth-date dependent critical level utilitarian objective function:

$$\int_{t=0}^{\infty} N_t e^{-\rho t} [u(c_t) - \alpha] dt \quad (1)$$

s.t.

$$\dot{A}_t = \bar{r}_t A_t + w_t N_t - c_t N - \tau_t^N N_t \quad (2)$$

where $u(c_t)$ is the instantaneous utility function, increasing and concave in c_t , $\rho > 0$ is the intergenerational discount rate and $\alpha > 0$ is the critical level. Since we fix neutrality consumption to zero (i.e. $u(0)=0$), this implies that c^α , satisfying $u(c^\alpha)=\alpha$, is strictly positive. Moreover, A_t is household wealth, $\bar{r}_t = r_t(1 - \tau_t^k)$ is net of tax interest rate, and τ_t^k and τ_t^N are the tax rate on capital income and on the population (household) size, respectively.

The population size, N_t , grows at rate n_t , i.e.

$$\frac{\dot{N}_t}{N_t} = n_t. \quad (3)$$

We assume that there are lower and upper bounds on the population growth rate: $n_t \in [\underline{n}, \bar{n}]$. Realistically, there is a physical constraint at each period of time on how many children a parent can have. There is also a constraint on how low the population growth can be. First, we do not allow individuals to be eliminated from the population (in that there is no axiomatic foundation for that). Moreover, even if nobody wants to reproduce there will always be accidental births. Clearly, from eq. (1) the problem has a finite solution only if $\rho > \bar{n}$ which we assume throughout our analysis.

2.2. Firms

Assuming constant-returns-to-scale production technology, $F(K_t, L_t)$, zero capital depreciation rate and perfect competition, firms hire capital, K , and labour services, L , (where $L_t \equiv l_t N_t = N_t$) on the spot market and remunerate them according to their marginal productivity, such that

$$F_{K_t} = r_t \quad (4a)$$

$$F_{L_t} = w_t. \quad (4b)$$

Moreover, the economy resource constraint is:

$$\dot{K}_t = F(K_t, L_t) - c_t N_t - g_t N_t. \quad (5)$$

2.3. The government

We allow government to finance an exogenous stream of public expenditure by issuing taxes, both on capital income and population size, and debt, B , whose law of motion is the following:

$$\dot{B}_t = r_t B_t + \tau^k r_t A_t - \tau_t^N N_t + N_t g_t. \quad (6)$$

We take g as exogenous (rather than $G=gN$), preserving second-best analysis as N grows. This is a natural assumption when population size is endogenous.

We should note a potential externality problem. If the government is fixing a stream of per capita public spending, the total expenditure will be proportional to the population size. When individual families decide on family sizes, they will not take into account the externality on the government's spending side. Consequently, a system of lump-sum taxation (lump-sum per family) will not implement the first-best (as mentioned before, however, in absence of government spending and taxation, the competitive equilibrium is Pareto-efficient).

2.4. Per-capita formulation

In some instances, it will be convenient to use per-capita notation. We then define the capital intensity $k \equiv \frac{K}{N}$, such that, by exploiting constant returns to scale in the production function we can write: $F(K, L) = Nf(k)$, $F_L(K, L) = f(k) - f'(k)k$. Hence, the capital and debt accumulation constraints in per capita terms can be written as:

$$\dot{k}_t = f(k_t) - c_t - n_t k_t - g_t \quad (5')$$

$$\dot{b}_t = (r_t - n) b_t - \tau^k r_t a_t - \tau_t^N + g_t, \quad (6')$$

where a_t is per-capita individual assets (A_t/N_t).

3. Decentralized solution

The problem of the individual (household) is to maximize (1) subject to (2) and $n_t \in [\underline{n}, \bar{n}]$, taking A_0 and N_0 as given. The current value Hamiltonian is:

$$H_t = N_t \left[u(c_t) - \alpha \right] + q_t \left[\bar{r}_t A_t + w_t N_t - c_t N_t - \tau_t^N N_t \right] + \lambda_t n_t N_t \quad (7)$$

The first order conditions are the following⁶:

$$\frac{\partial H}{\partial c} = 0 \Rightarrow u' = q \Rightarrow \dot{c} = (\rho - \bar{r}) \frac{u'}{u''} \quad (7a)$$

$$\frac{\partial H}{\partial A} = \rho q - \dot{q} \Rightarrow \dot{q} = (\rho - \bar{r}) q \quad (7b)$$

$$\frac{\partial H}{\partial N} = \rho \lambda - \dot{\lambda} \Rightarrow \dot{\lambda} = (\rho - n) \lambda - (u - \alpha) - q[w - c - \tau^N] \quad (7c)$$

$$\frac{\partial H}{\partial n} = \lambda N \begin{matrix} > \\ < \end{matrix} 0 \quad (7d)$$

and the transversality conditions are

$$\lim_{t \rightarrow \infty} e^{-\rho t} q_t A_t = 0, \quad \lim_{t \rightarrow \infty} e^{-\rho t} \lambda_t N_t = 0. \quad (7e)$$

We now characterize the competitive equilibrium. Supposing that the economy starts at time $t=0$, we recall that a competitive equilibrium is time paths of: a) policies

$$\pi = \{ \tau^k(t), \tau^N(t), B(t) \}_0^\infty, \quad \text{b) allocations} \quad \varepsilon = \{ c(t), N(t), K(t) \}_0^\infty, \quad \text{c) prices}$$

$\theta = \{ w(t), r(t) \}_0^\infty$, such that, at each point in time t : b) satisfy max eq. (1) subject to eqs. (2) and (3), given a) and c); c) satisfies eqs. (4a), (4b) and eqs. (5') and (6') are satisfied. Moreover, under a competitive equilibrium, the Walras law holds, such that the following condition applies:

$$a_t = k_t + b_t. \quad (8)$$

We first examine the nature of the population choice, n . Since λ (the co-state for N) is the shadow value of population size, from equation (7d) we can see that if λ is different from zero, either population should be increased as much as possible ($\lambda > 0$), or as little as possible ($\lambda < 0$).

In fact, by integrating (7c) we get:

$$\lambda_t = \int_t^\infty e^{-\int_t^\tau (\rho - n_s) ds} \left[u(c_\tau) - \alpha + u'_\tau (F_{L_\tau} - c_\tau - \tau^N) \right] d\tau \begin{matrix} > \\ < \end{matrix} 0. \quad (9)$$

The integrand is the difference between two terms. One term, $u(c) - \alpha + u' F_L$, is the value (in utility units) a new individual brings to the family (his/her utility in excess of the critical level α plus the utility value of his/her labour endowment), and the other is the value (in utility units) of what the new individual is taking out of the family (consumption

⁶ We omit the subscript referring to time when it causes no ambiguity to the reader.

plus the population tax). If these terms are the same for the entire future, then population size is optimal, and λ is zero. However, we have shown in another work that, without taxes, along the dynamic path, population will grow either at the maximum or at the minimum speed, the interior solution arising only at the steady state (see Renström and Spataro 2010). We will briefly discuss the dynamic properties of the model in the next section.

3.1. Steady state

If the steady state solution for n is interior, then $\lambda = 0$ and by exploiting eqs. (4a)-(6) and (7a)-(7e) we can provide the following three equations which fully characterize the steady state:

$$f'(k^{ss})(1 - \tau^k) = \rho \quad (10a)$$

$$f(k^{ss}) - n^{ss}k^{ss} - g = c^{ss} \quad (10b)$$

$$\Psi = 0: \frac{u(c^{ss}) - \alpha}{u'(c^{ss})} = c^{ss} + \tau^N - [f(k^{ss}) - f'(k^{ss})k^{ss}]. \quad (10c)$$

Moreover, it can be shown that the interior solution for n is granted by the critical level belonging to an open, positive interval (for details see Renström and Spataro 2010):

$$\alpha \in (\bar{\alpha}, \underline{\alpha}), \quad (11)$$

where

$$\bar{\alpha} \equiv u(\bar{c}) - u'(\bar{c})(\rho - \bar{n})a^{ss} \quad (11a)$$

$$\underline{\alpha} \equiv u(\underline{c}) - u'(\underline{c})(\rho - \underline{n})a^{ss} \quad (11b)$$

$$\text{and } \bar{c} \equiv F_N^{ss} + (F_K^{ss} - \bar{n})k^{ss} - g \text{ and } \underline{c} \equiv F_N^{ss} + (F_K^{ss} - \underline{n})k^{ss} - g.$$

In fact, when $\alpha = 0$ (i.e. Classical Utilitarianism case) or $0 < \alpha \leq \bar{\alpha}$, then the solution for the population growth rate will be $n^{ss} = \bar{n}$, that is the repugnant conclusion would arise, in that the population should grow at the maximum speed; on the other hand, if $\alpha \geq \bar{\alpha}$ then the solution would be the opposite, $n^{ss} = \underline{n}$, that is the population growth rate should be at its minimum, which resembles the solution obtained in the Average Utilitarianism case.

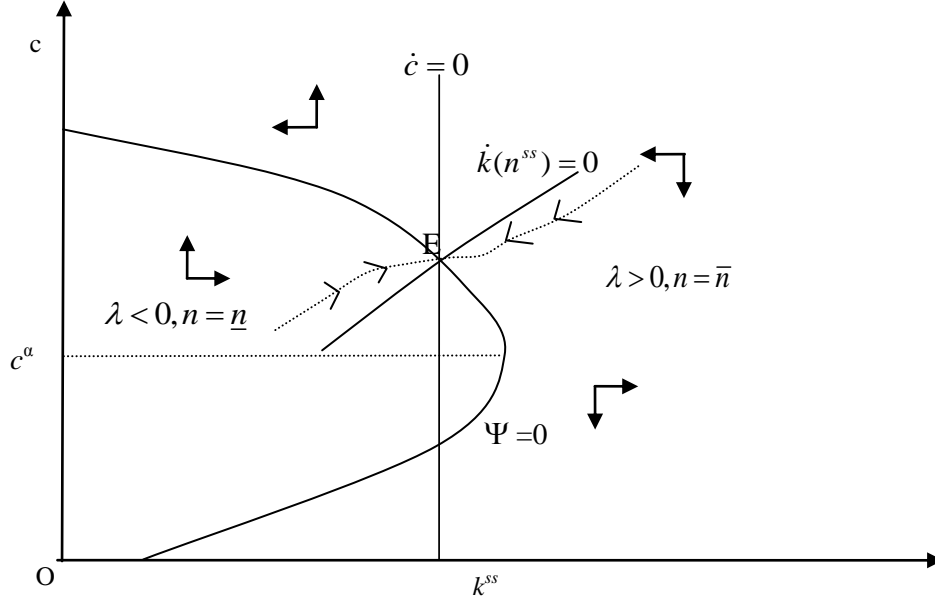
We will assume that α is in the interval given by (11), such that the solution entails an interior value for n . If this were not the case, our model would resemble a Cass-Koopmans-Ramsey model with exogenous fertility, which has been already deeply studied.

In Figure 1 we depict the three loci described by eqs. (10a)-(10c) where the steady state equilibrium is represented by point E. Equation (10c) gives all combinations of per-capita steady state consumption and per-capita capital that constitute an optimal population size. As anticipated above, this is the case when what an individual brings to the family (utility above α plus the labour endowment) is equal to what he/she takes out (consumption plus the population tax). These combinations are depicted by the $\Psi = 0$ locus in Figure 1. For trajectories inside the $\Psi = 0$ locus, λ is negative and consequently n is at its lower corner

\underline{n} . For trajectories outside the $\Psi = 0$ locus, λ is positive and n is at its higher corner \bar{n} . The steady state value of per-capita capital is given by $\rho = f'(k^{ss})(1 - \tau^k)$, giving the vertical $\dot{c} = 0$ line. The steady state population growth rate is such that $\dot{k} = 0$ line cuts in point E. We should notice that the trajectories leading to E are not the usual saddle-paths. The reason is that we are in a corner with respect to n along the transition. For capital stocks lower than k^{ss} it is optimal to pick an unstable trajectory in a system where $n = \underline{n}$ and when reaching E, switching from \underline{n} to n^{ss} . Similarly, for capital stocks greater than k^{ss} it is optimal to take an unstable trajectory in a system when $n = \bar{n}$ and when reaching E, let n jump from \bar{n} to n^{ss} . Point E is also reached in finite time.⁷

We will show in section 5 that under the optimal tax programme (in the first or the second-best) that the steady state per capita assets, a^{ss} , are positive. Consequently, the steady state consumption level is greater than the one giving critical-level utility, i.e. $c^{ss} > c^a$. To show the latter it is sufficient to see that, by substituting eqs. (4a) and (10a) into the steady state equation for the household budget constraint (eq. 2), expressed in per-capita terms, the RHS of eq. (10c) is equal to $(\rho - n^{ss})a^{ss} > 0$, where the inequality follows from $\rho > \bar{n} \geq n^{ss}$ and, hence, by eq. (10c) we have $u(c^{ss}) > \alpha$.

Figure 1: The steady state equilibrium



4. Positive analysis of taxation

In the section we aim at analysing the effects of taxation on the equilibrium of our economy. We will perform some comparative statistics exercises in which we either let one tax change, keeping the other constant while adjusting public debt, or let both taxes change simultaneously so as to keep the steady state per capita public debt level the same.

Since in the normative analysis in section 5 individual assets are shown to be positive, we will confine our positive analysis to the cases in which such a feature holds.

As for the effects of a change of the tax on the family size, the results are summarized by the following Proposition:

⁷ For details see Renström and Spataro (2010).

Proposition 1: *At the steady state, an increase of the tax on the family size increases consumption, decreases the rate of growth of population and leaves capital intensity unchanged.*

Proof: Since eq. (10a) provides the solution for $k^{ss} = k^{ss}(\tau^k)$ which, under concavity of the production function, implies

$$\frac{dk}{d\tau^k} = \frac{f'}{f''(1-\tau^k)} < 0. \quad (12)$$

By substituting such a solution into (10b) and (10c) we obtain the expressions for the solutions of n and c as functions of the taxes:

$$f(k^{ss}(\tau^k)) - n^{ss}(\tau^k, \tau^N) k^{ss}(\tau^k) - g = c^{ss}(\tau^k, \tau^N) \quad (10b')$$

$$\frac{u(c^{ss}(\tau^k, \tau^N)) - \alpha}{u'(c^{ss}(\tau^k, \tau^N))} = c^{ss}(\tau^k, \tau^N) + \tau^N - [f(k^{ss}(\tau^k)) - f'(k^{ss}(\tau^k)) k^{ss}(\tau^k)]. \quad (10c')$$

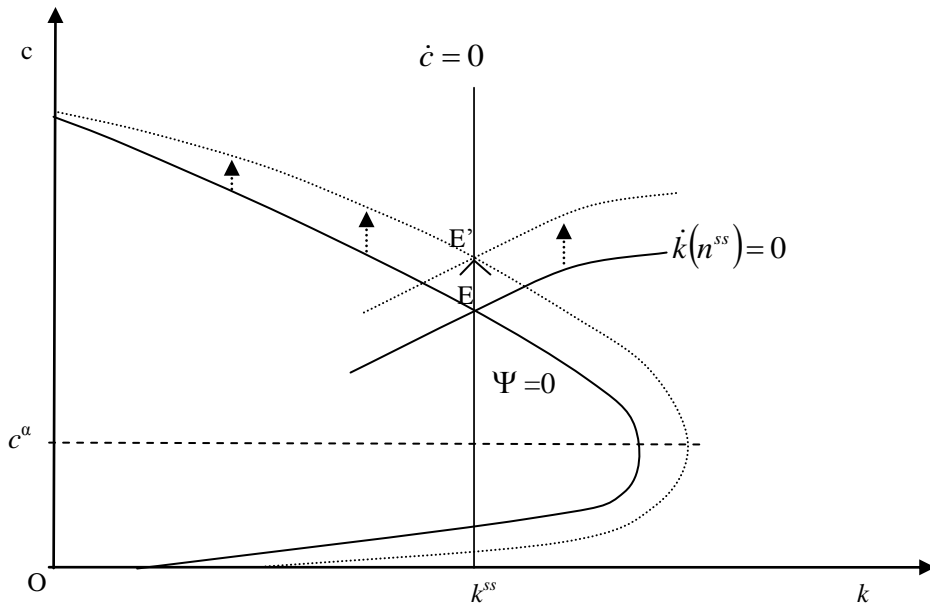
Hence, by totally differentiating eqs. (10b') and (10c') with respect to τ^N we get that at the steady state

$$\frac{\partial n}{\partial \tau^N} = -\frac{1}{k} \frac{\partial c}{\partial \tau^N} \quad (13a)$$

$$\frac{\partial c}{\partial \tau^N} = \frac{u'^2}{-u''(u-\alpha)} \begin{matrix} > \\ < \end{matrix} 0 \text{ if } u \begin{matrix} > \\ < \end{matrix} \alpha. \quad (13b)$$

Since at the steady state $u > \alpha$, the Proposition is proved. ■

Figure 2: The effects of an increase of the tax on the population size



Proposition 1 is illustrated in Figure 2. When the population tax is increased (keeping the capital-income tax constant), the $\Psi = 0$ locus shifts outwards (i.e. the region for which $n_t = \underline{n}$, is expanded). As a consequence the new steady state is where the $\dot{c} = 0$ line cuts the new $\Psi = 0$ locus, at E' , and the new steady state growth rate for population is lower than previously ($\dot{k} = 0$ shifts upwards). The new steady state level of consumption is higher, while the capital intensity is unaffected. If this policy comes as a surprise tax change for the individual family, per-capita consumption jumps from E to E' , and the population growth rate falls to the new level immediately. Consequently there is no transition dynamics in this case.

As for the effects of a change of the capital income tax we can provide the following Proposition:

Proposition 2: *At the steady state, an increase of the capital income tax increases consumption and decreases both the rate of growth of population and capital intensity.*

Proof: By differentiating eqs. (10b') and (10c') with respect to τ^k we get:

$$\frac{\partial c}{\partial \tau^k} = \frac{f'' k (u')^2}{-u''(u - \alpha)} \frac{dk}{d\tau^k} > 0 \quad (14a)$$

$$\text{and } \frac{\partial n}{\partial \tau^k} = \frac{1}{k} \left[(f' - n) \frac{dk}{d\tau^k} - \frac{\partial c}{\partial \tau^k} \right].$$

Given that at the steady state $f' = \frac{\rho}{(1 - \tau^k)}$ and, moreover, $\rho > n$, it follows that, under nonnegative capital taxation, $f' > n$ and, hence,

$$\frac{\partial n^{ss}}{\partial \tau^k} = \left[(f'^{ss} - n^{ss}) \frac{dk^{ss}}{d\tau^k} - \frac{\partial c^{ss}}{\partial \tau^k} \right] < 0 \quad \blacksquare \quad (14b)$$

Figure 3: The effects of an increase of the capital income tax

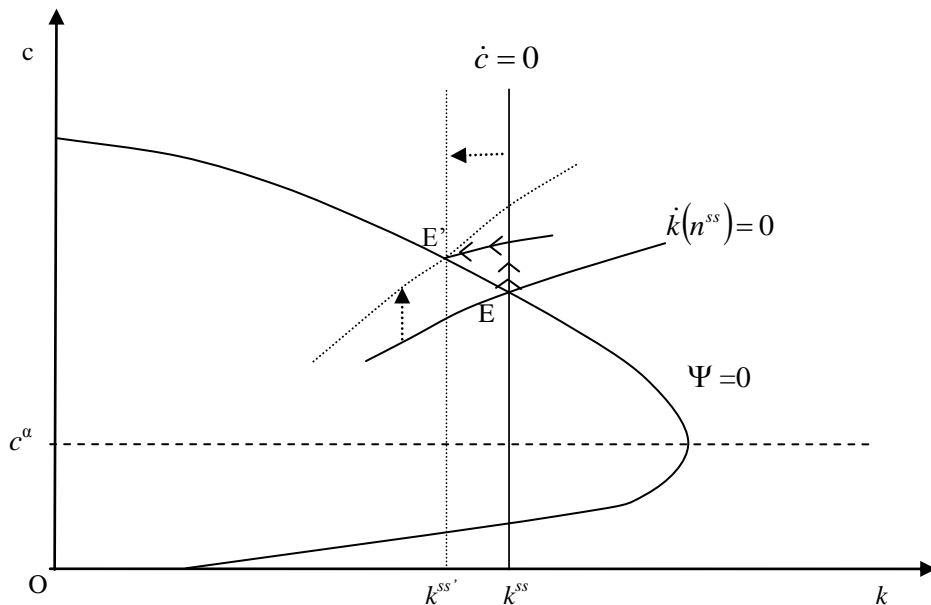


Figure 3 illustrates the result summarized in Proposition 2. Note that, in this case, when the capital income tax increases, the $\dot{c}=0$ shifts to the left and the $\dot{k}=0$ moves up, while the $\Psi=0$ does not change, such that the new steady state equilibrium moves from point E to point E'. The latter entails a higher consumption level and both lower population growth rate and lower capital intensity.

If this tax change comes as a surprise, per capita consumption jumps onto the new trajectory leading to E'. This implies that per-capita consumption first jumps to a high level, and then gradually falls to its new level, creating a consumption boom. During the transition, the economy is outside the $\Psi=0$ locus and consequently population growth is at its maximum, \bar{n} . When reaching the new steady state in finite time population growth falls to its lower new steady state value. Thus, the economy experiences a population growth burst ("baby boom") and then a fall in the population growth rate.

As a general comment on the analysis carried out so far, we can say that the long-run effects on the economy of an increase of either taxes are very similar, in that both reduce the population growth and increase consumption. However, the increase of the capital income tax creates temporary population and consumption bursts and reduces the steady state capital stock, while an increase in the population tax does not.

Finally, we analyze the case in which the government changes both taxes in such a way that per capita debt remains constant. Since the changes in the capital and the population taxes have the same qualitative effects, if we were to increase one of them and decrease the other so as to keep the debt level constant, we may ask which tax dominates.

Preliminarily, we provide a sufficient condition according to which any such policy implies that taxes move in opposite directions (e.g. an increase of the capital income tax with constant per capita debt implies a reduction of the tax on population size).

Lemma 1: *At the steady state, an increase (decrease) of capital income tax aiming at maintaining per-capita debt constant, implies a reduction (increase) of the tax on the population size if the capital income tax is lower than a threshold, i.e.:*

$$\frac{d\tau^N}{d\tau^k} < 0 \text{ if } \tau^k < \bar{\tau}^k, \text{ where } \bar{\tau}^k = -\left[\frac{f''k}{f'}(1+M) + \frac{b}{k} \frac{f'-n}{f'} \right] \text{ and } M \equiv \frac{b}{k} \frac{u'^2}{u''(u-\alpha)}.$$

Proof: By exploiting eqs. (8) and (10a), the steady state government budget constraint can be written as:

$$b(\rho - n) = k(f' - \rho) + \tau^N - g;$$

totally differentiating the above expression with respect to τ^k yields:

$$\frac{dn}{d\tau^k} = -\frac{1}{b} \frac{d\tau^N}{d\tau^k} + \frac{1}{b} (\rho - f' - f''k) \frac{dk}{d\tau^k}. \text{ Recalling that } \frac{dn}{d\tau^k} = \frac{\partial n}{\partial \tau^N} \frac{d\tau^N}{d\tau^k} + \frac{\partial n}{\partial \tau^k}, \text{ and exploiting eqs. (13a) and (14b) we get also:}$$

$$\frac{dn}{d\tau^k} = \frac{1}{k} \frac{(u')^2}{u''(u-\alpha)} \frac{d\tau^N}{d\tau^k} + \frac{1}{k} \left[f' - n + \frac{f''k(u')^2}{u''(u-\alpha)} \right] \frac{dk}{d\tau^k}. \quad (15)$$

Hence, by equating the two above expressions for $\frac{dn}{d\tau^k}$ and collecting terms it follows that:

$$\frac{d\tau^N}{d\tau^k} = f' \left(-\frac{\tau^k}{1+M} - \frac{f''k}{f'} - \frac{b}{k} \frac{f'-n}{f'} \frac{1}{1+M} \right) \frac{dk}{d\tau^k} \quad (16)$$

where $M \equiv \frac{b}{k} \frac{u^2}{u''(u-\alpha)}$. Let us assume that $(1+M)$ is positive (by continuity, at least around $b=0$ this condition is satisfied). Since $\frac{dk}{d\tau^k} < 0$, it follows that

$$\frac{d\tau^N}{d\tau^k} < 0 \Leftrightarrow \tau^k < \bar{\tau}^k. \quad \blacksquare$$

$\bar{\tau}^k$ in Lemma 1 is the steady-state Laffer maximum capital-tax rate. For any initial capital tax rate lower than this level, when the government is increasing the capital tax rate, it can lower the population tax rate and keep the debt level the same. If the initial capital tax rate is higher than the Laffer maximum, then an increase in the rate makes the government to lose revenue, and to maintain the same level of debt, it would have to increase the population tax rate. In fact, in the latter case, there is room for decreasing both taxes. We will assume that the initial capital tax rate is lower than the Laffer maximum, i.e. that $\tau^k < \bar{\tau}^k$ ⁸.

We now focus on the sign of the derivatives of both c and n , which, a priori and differently from the effect on the capital intensity, are ambiguous. Our findings are summarized by the following Proposition 3:

Proposition 3: *At the steady state, a tax reform consisting in an increase (decrease) of the capital income tax and a reduction (increase) of the tax on the population size in such a way to leave per-capita debt unchanged, implies that both capital intensity and the population growth rate decrease (increase) and per capita consumption increases (decreases).*

Proof: By plugging eq. (16) into eq. (15) and collecting terms it descends that

$$\frac{dn}{d\tau^k} = \frac{1}{1+M} \left[\frac{f'-n}{k} + \frac{M}{b} (\rho - f') \right] \frac{dk}{d\tau^k} < 0. \quad (17a)$$

Moreover, by differentiating (10c) with respect to τ^k we get that

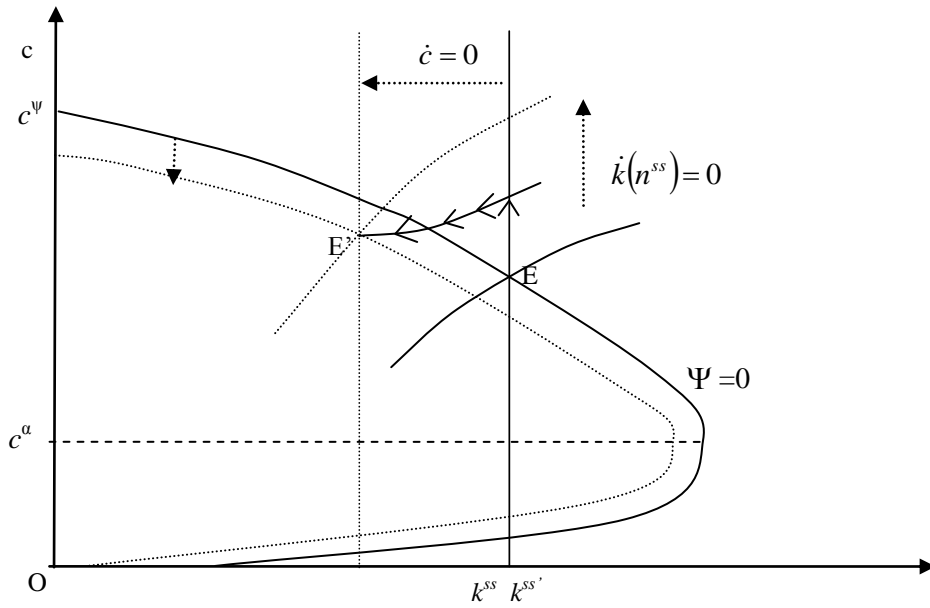
$$\begin{aligned} \frac{dc}{d\tau^k} &= (f'-n) \frac{dk}{d\tau^k} - \frac{dn}{d\tau^k} > 0 \text{ and, more precisely,} \\ \frac{dc}{d\tau^k} &= \frac{k}{b} \frac{M}{1+M} \left((f'-n) \frac{b}{k} + f' - \rho \right) \frac{dk}{d\tau^k} > 0. \quad \blacksquare \end{aligned} \quad (17b)$$

⁸ Incidentally, note that when $b=0$, the condition above boils down to $\tau^k < -\frac{f''k}{f'}$ (and the latter inequality is

both necessary and sufficient for $\frac{d\tau^N}{d\tau^k} < 0$).

In Figure 4 we illustrate the above Proposition. The increase in the capital income-tax makes the $\dot{c}=0$ line shift to the left, and the reduction in the population tax makes the $\Psi=0$ locus move left (shrinking the region for which n is at its lowest corner, \underline{n}). The new steady state is at point E' , associated with a lower population growth rate (since the $\dot{k}=0$ line shifts upwards), higher per-capita consumption, and lower capital intensity. For a surprise tax reform of this kind (keeping the new steady state government debt level the same), per-capita consumption first jumps to the new trajectory, and then gradually falls toward its new steady state level. Since the economy is outside the $\Psi=0$ locus during the transition, the population growth rate is at its maximum. When the economy reaches steady state in finite time, the population growth rate falls to its new lower value. The dynamic path is qualitatively the same as in Figure 3.

Figure 4: The effects of constant per-capita debt redistribution of taxes



The latter tax reform showed that the effect through the capital income tax dominates.

5. The Ramsey problem

We now solve the optimal tax problem (Ramsey problem). We shall first find the first-best solution, and then move on to the second-best. Since the first-best is obtained as a solution to the second-best problem, when the second-best constraints do not bind, we formulate the latter problem from the outset. In doing so, we adopt the primal approach, consisting of the maximization of a direct social welfare function through the choice of quantities (i.e. allocations; see Atkinson and Stiglitz 1972)⁹. For this purpose it is necessary to restrict the set of allocations among which the government can choose to those that can be decentralized as a competitive equilibrium. We first find the constraints that must be imposed on the government's problem in order to comply with this requirement.

In our framework there are two implementability constraints, one associated with the individual family's intertemporal consumption choice and one associated with the fertility choice.

⁹ On the contrary, the dual approach takes prices and tax rates as control variables. For a survey see Renström (1999).

The first constraint is the individual budget constraint with prices substituted for by using the consumption Euler equation (the formal derivation is provided in Appendix A.1):

$$A_0 u'_0 = - \int_0^{\infty} e^{-\rho t} u'_t (w_t - c_t - \tau_t^N) N_t dt \quad (18a)$$

The second one is given by the following:

$$n_t = \begin{cases} \bar{n}, \lambda > 0 \\ n_t^*, \lambda = 0 \\ \underline{n}, \lambda < 0 \end{cases} \quad (18b)$$

where λ is according to (9), and n_t^* is any level of n .

Finally there are the feasibility constraints, one which requires that private and public consumption plus investment be equal to aggregate output (eq. 5); the other is given by eq. (3).

5.1. Solution

In this section we characterize the solution to the Ramsey problem. As already mentioned, the policymaker has to abide both the implementability and the feasibility constraints in order to insure that the optimal allocation, solution of the Ramsey problem, implements a competitive equilibrium.

Hence, supposing that the policy is introduced in period 0, the problem of the policymaker to maximize (1) subject to eq. (18a), and, $\forall t \geq 0$, eqs. (5), (3) and (18b).

Note that the latter constraint (18a), involving λ (eq. 9), entails both an integral and an inequality, which is difficult to be dealt with. However, as already mentioned, for trajectories inside (outside) the $\Psi = 0$ locus, the expression in square brackets in equation (9) must be negative (positive) at each instant t (for details see Renstrom and Spataro 2010). We associate this latter inequality with the multiplier ω . Hence, the current value Hamiltonian is:

$$H_t = N_t [u(c_t) - \alpha] + \mu [-u'_t (F_{L_t} - c_t - \tau_t^N) N_t] - \omega_t N_t [u(c_t) - \alpha + u'_t (F_{L_t} - c_t - \tau_t^N)] + \gamma_t [F_t - c_t N_t - g_t N_t] + \phi_t n_t N_t \quad (19)$$

First order conditions for this problem are the following (we omit the time subscript when it does not cause ambiguity to the reader and the transversality conditions for the sake of brevity):

$$\frac{\partial H}{\partial c} = 0 \Rightarrow u'(1 + \mu) - u''(F_L - c - \tau^N)(\mu + \omega) = \gamma \quad (19a)$$

$$\frac{\partial H}{\partial K} = \rho\gamma - \dot{\gamma} \Rightarrow \dot{\gamma} = \gamma(\rho - F_K) + u' F_{LK}(\mu + \omega) \quad (19b)$$

$$\frac{\partial H}{\partial N} = \rho\phi - \dot{\phi} \Rightarrow \dot{\phi} = (\rho - n)\phi - (u - \alpha) + \mu u'(F_L - c - \tau^N) + \gamma(F_L - c - g) + u'(\mu + \omega)F_{LL}N \quad (19c)$$

$$\frac{\partial H}{\partial n} = \phi N \begin{matrix} > \\ < \end{matrix} 0 \quad (19d)$$

where in eq. (19c) we omit the term $\omega[u - \alpha + u'(F_L - c - \tau^N)]$ because this condition, being a Kuhn Tucker complementary slackness condition, must be identically equal to zero. Recall that γ , being the shadow price of capital, is strictly positive and μ , being a measure of the deadweight loss stemming from distortionary taxation, is zero at the first-best and positive at the second-best.

Moreover, equating eqs. (7a) and eq. (19b) to zero, yields:

$$\tau^k = \frac{(\mu + \omega)u'F_{LK}}{F_K\gamma}. \quad (20)$$

In light of the results above, we can now characterize the first-best policy.

Proposition 4: *The first-best policy implies that capital income tax be zero, the tax on the family size be equal to the per-capita public expenditure and the public debt be equal to zero.*

Proof: At the first-best the government controls c, n, k , directly. Consequently A and λ are not binding, which implies that $\mu = \omega = 0$. By (19a), (19b) and (7a), (7b), at each instant t $\gamma = q = u'$ and $\tau^k = 0$. Moreover, since the first-best $\lambda = \phi$ (i.e. the government evaluation of the population is equal to the households' evaluation), from (7c) and (19c) it follows that $\tau^N = g$. Finally, since $\gamma = q$ (the marginal value of capital is equal to marginal value of private assets), it descends that, at each instant t , $a=k$ and $b=0$. ■

A comment on the latter result is worth making. The reason why the population tax implements the first-best rather than a family-level lump-sum tax, is because the externality a family has on the government budget when choosing the number of children is perfectly internalised when $\tau^N = g$. If there is any public debt it should be defaulted upon, otherwise the population tax would have to exceed the public expenditure level, and the first best would not be implemented.

Suppose now that the first-best taxation is not implementable; more precisely, we assume that the constraint $\tau^N \leq \tau^N_{\max} < g$ is binding which happens if

$$\frac{\partial H}{\partial \tau^N} = u'(\mu + \omega)N > 0, \quad (21)$$

which means that the Hamiltonian is increasing in the population tax as long as the second-best constraint binds.

In this situation, only a second-best allocation is implementable, with the level of capital income tax given by (20). Hence, we can summarise our finding as follows:

Proposition 5: *The second-best tax structure implies $\tau^N = \tau_{\max}^N < g$ and positive capital income $\tau^k = \frac{(\mu + \omega)\mu' F_{LK}}{F_K \gamma} > 0$. Moreover, the optimal level of debt is negative.*

Proof: The levels of taxes, τ^N and τ^k , descend by construction and by eq. (20) respectively. Hence we focus on the sign and the level of optimal debt. By (5) and (19b) we get

$$\frac{d(K\gamma)}{dt} = \rho K\gamma + \gamma(f - c - g)N + F_{LK} K N u'(\mu + \omega)$$

and eqs. (3) and (19c) yield:

$$\frac{d(\phi N)}{dt} = \rho \phi N - [u(c) - \alpha]N + \mu u' N (F_L - c - \tau^N + F_{LL} N) + u' \omega F_{LL} N^2 - \gamma(F_N - c - g),$$

such that, by exploiting CRS, whereby $F_{LK} K = -F_{LL} N$

$$\frac{d(K\gamma)}{dt} + \frac{d(\phi N)}{dt} = \rho(K\gamma + \phi N) - (u - \alpha)N + \mu N u' (F_L - c - \tau^N).$$

Finally, by integrating both sides it follows that:

$$e^{-\rho(T-t_0)} (\phi_{t_T} N_{t_T} + K_{t_T} \gamma_{t_T}) - \phi_{t_0} N_{t_0} - K_{t_0} \gamma_{t_0} = - \int_{t_0}^T e^{-\rho(t-t_0)} N [u - \alpha - \mu u' (F_N - c - \tau^N)] dt.$$

Taking the limit for $T \rightarrow \infty$ and indicating t_1 as the instant in which the steady state is reached (recall that this economy reaches the steady state in finite time), whereby both ϕ and λ are equal to zero, exploiting transversality conditions we end up with the following expression:

$$K_{t_1} \gamma_{t_1} = (1 + \mu) \int_{t_1}^{\infty} e^{-\rho(t-t_1)} N u' [c + \tau^N - F_N] dt.$$

Next, by eq (18a) the integral at the RHS of the equation above is equal to $u'_{t_1} A_{t_1}$, which yields

$$(1 + \mu) = \frac{K_{t_1} \gamma_{t_1}}{u'_{t_1} A_{t_1}}.$$

Note that the equation above states that, at the steady state of the second-best, private assets are strictly positive. By plugging the expression above for γ_{t_1} into (19a) (for any generic instant $t \geq t_1$), substituting from (20) for $(\mu + \omega)$ and exploiting (8), the following equality holds:

$$u' \frac{K\gamma}{u'A} - u''(F_L - c - \tau^N)(\mu + \omega) = \gamma \Rightarrow \gamma \frac{K-A}{A} - u''(F_L - c - \tau^N) \gamma \frac{\tau^k F_K}{F_{LK} u'} = 0.$$

Finally, recalling that at the steady state equilibrium $-(F_L - c - \tau^N) = (\rho - n)a$, we get

$$\frac{b}{a} = u''(\rho - n)k\gamma \frac{\tau^k F_K}{F_{LK} u'} < 0. \quad \blacksquare$$

A final comment on the results is worth making. The nonzero capital income tax is non-standard in the traditional literature on optimal taxation and exogenous population growth, in that, typically, in the long run the second-best result entails zero tax on capital income, stemming from the optimality of uniform commodity taxation (Atkinson and Stiglitz 1972), although some exceptions may arise¹⁰. The rationale of our result is the following: when labour supply is exogenous there are labour rents present. If those rents are not taxed at 100%, the standard second-best results will not hold, in particular results on uniform commodity taxation. In fact, a capital tax partially taxes those rents (because $F_{LK} > 0$).

As for the negative level of debt in the steady state, under the second-best it is optimal to run primary surpluses at the beginning of the tax programme, arriving at the steady state with public assets. At the steady state, tax receipts fall below the level of public expenditure, though not being zero (i.e. it is still optimal to carry tax burden to the steady state).

5.2. Endogenous labour supply

We now show the solution to the Ramsey problem when individuals can endogenously offer their labour services and (distortionary) taxes on wages are levied. The instantaneous utility function is now of the form $u(c_t, l_t)$, assumed to be decreasing in labour supply l_t and is strictly concave. Total labour supply is then $N_t l_t = L_t$. The household budget constraint is now

$$\dot{A}_t = \bar{r}_t A_t + \bar{w}_t l_t N_t - c_t N_t - \tau_t^N N_t \quad (2')$$

where $\bar{w} = w(1 - \tau')$ is the wage rate net of labour income tax τ' , and the first order conditions of the individual problem entail now the following condition:

$$\frac{\partial H}{\partial l} = 0 \Rightarrow u_l = -\bar{w}q, \quad (7a_bis)$$

which, combined with eq. (7a) provides the following:

$$\frac{u_l}{u_c} = -\bar{w}. \quad (7a_ter)$$

¹⁰ For example, in OLG economies (as argued by Erosa and Gervais 2002) or in presence of different discounting between government and individuals (see De Bonis and Spataro 2005) or a combination of two (see Spataro and De Bonis 2008).

Moreover, recall that the decentralized equilibrium implies that the gross wage rate be equal to the marginal productivity of labour, that is

$$F_{L_t} = w_t \quad (4b')$$

All this said, the problem of the policymaker becomes:

$$\max \int_{t=0}^{\infty} N_t e^{-\rho t} [u(c_t, l_t) - \alpha] dt$$

subject to

$$A_0 u'_0 = \int_0^{\infty} e^{-\rho t} [u_{l_t} l_t + u_{c_t} (c_t + \tau^N)] N_t dt,$$

and eqs. (5), (3) and (18b), where λ is given by the following expression:

$$\lambda_t = \int_t^{\infty} e^{-\int_t^{\tau} (\rho - n_s) ds} [u(c_{\tau}, l_{\tau}) - \alpha + q_{\tau} (\bar{w}_{\tau} l_{\tau} - c_{\tau} - \tau^N)] d\tau \begin{matrix} \geq \\ < \end{matrix} 0.$$

Again, it can be shown that, for trajectories inside (outside) the $\Psi=0$ locus, the expression in brackets in the integral above must be negative (positive) in each instant t . Hence, by making use of eqs. (7a) and (7a_ter), the current value Hamiltonian function is:

$$H_t = N_t [u(c_t, l_t) - \alpha] + \mu [u_{l_t} l_t + u_{c_t} (c_t + \tau^N)] N_t - \omega_t N_t [u(c_t, l_t) - \alpha - u_{l_t} l_t - u_{c_t} (c_t + \tau^N)] + \gamma_t [F_t - c_t N_t - g_t N_t] + \phi_t n_t N_t \quad (22)$$

Then, FOCs now imply:

$$\frac{\partial H}{\partial c} = 0 \Rightarrow u_c [1 + \mu + (\mu + \omega) \Delta_c] = \gamma \quad (22a)$$

$$\frac{\partial H}{\partial l} = 0 \Rightarrow u_l [1 + \mu + (\mu + \omega) \Delta_l] = -\gamma F_L \quad (22b)$$

$$\frac{\partial H}{\partial K} = \rho \gamma - \dot{\gamma} \Rightarrow \dot{\gamma} = (\rho - F_K) \gamma \quad (22c)$$

$$\frac{\partial H}{\partial N} = \rho \phi - \dot{\phi} \Rightarrow \dot{\phi} = (\rho - n) \phi - (u - \alpha)(1 - \omega) - (\mu + \omega) [u_l l + u_c (c + \tau^N)] - \gamma (F_L - c - g) \quad (22d)$$

$$\frac{\partial H}{\partial n} = \phi N \begin{matrix} \geq \\ < \end{matrix} 0 \quad (22e)$$

where $\Delta_c \equiv \frac{u_{cc}(c + \tau^N) + u_{lc}l}{u_c}$ and $\Delta_l \equiv \frac{u_{cl}(c + \tau^N) + u_{ll}l}{u_l}$ are usually referred to as the “general equilibrium elasticity” of consumption and leisure, respectively. By dividing eq. (22b) by (22a) we obtain:

$$\frac{u_l [1 + \mu + (\mu + \omega)\Delta_l]}{u_c [1 + \mu + (\mu + \omega)\Delta_c]} = -F_L, \text{ and, using eq. (7a_ter) yields:}$$

$$\tau^l = \frac{(\mu + \omega)(\Delta_l - \Delta_c)}{[1 + \mu + (\mu + \omega)\Delta_c]}. \quad (23)$$

As for capital income tax, since at the steady state the LHS of (22a) is constant, then the RHS is constant as well, such that, by eq. (22c) it turns out that $\tau^k = 0$. Finally, as for τ^N , we can start by observing that the tax structure $\tau^k = 0$, $\tau^l = 0$, $\tau^N = g$ and $b=0$ would implement the first-best allocation. Hence, in order to get a second-best allocation, we again impose that the constraint $\tau^N \leq \tau^N_{\max} < g$ is binding, which is insured by the condition (21), assumed to hold.

Hence, we can provide the following proposition:

Proposition 6: *At the steady state the second-best tax structure implies that capital income tax be zero, the tax on the family size be equal to the maximum positive level τ^N_{\max} and labour income tax be nonzero. Sufficient condition for the latter tax to be positive is that leisure is non-inferior.*

Proof: Since the other results are clear-cut, here we provide the proof for the labour income tax. Recall that

$$\Delta_l - \Delta_c = -\left(\frac{u_{cc}}{u_c} - \frac{u_{cl}}{u_l}\right)(c + \tau^N) + \left(\frac{u_{ll}}{u_l} - \frac{u_{lc}}{u_c}\right)l \quad (24)$$

Preliminarily, if leisure is non-inferior, then $\frac{dl}{d\tau^N} \geq 0$. By differentiating per-capita budget constraint with respect to τ^N , exploiting eqs. (7a) and (7a_bis) one gets:

$$\frac{dl}{d\tau^N} = -\frac{u_{cl}}{u_c} \frac{dc}{d\tau^N} \geq 0 \Rightarrow -\left(\frac{u_{cc}}{u_c} - \frac{u_{cl}}{u_l}\right) \geq 0.$$

Moreover, by eq. (2'), expressed in per capita terms, at the steady state we get that:

$c + \tau^N = \bar{w}l + (\rho - n)a$. Hence, eq. (24) can be written as:

$$\Delta_l - \Delta_c = -\left(\frac{u_{cc}}{u_c} - \frac{u_{cl}}{u_l}\right)(\rho - n)a + \frac{l}{u_l} \left[u_{ll} - \frac{u_l}{u_c} u_{lc} + u_{cc} \left(\frac{u_l}{u_c} \right)^2 - \frac{u_l}{u_c} u_{cl} \right] \quad (24')$$

Since, by following the same steps as those made in the Proof of Proposition 5 it is possible to show that individual assets are positive (the complete proof is available from request to the authors) and given that at the steady state $\rho - n > 0$, it follows that the first term on the

RHS of eq. (24) is non-negative. As for the second term of the RHS, the term in square brackets is a quadratic form in the Hessian to $u(c, l)$. Since u is concave, the Hessian is negative definite and the quadratic form is negative. Consequently the last term:

$$\frac{l}{u_l} \begin{bmatrix} 1 & -\frac{u_l}{u_c} \end{bmatrix} \begin{bmatrix} u_{ll} & u_{cl} \\ u_{cl} & u_{cc} \end{bmatrix} \begin{bmatrix} 1 \\ -\frac{u_l}{u_c} \end{bmatrix}$$

is positive (given that $\frac{l}{u_l} < 0$) and $\Delta_l - \Delta_c \geq 0$. ■

As a final comment, we conclude that with endogenous labour supply, there are no labour rents, and with (yet distortionary) labour income taxation, the zero capital income tax result is restored (i.e. the optimality of uniform commodity taxation). Also, the labour income tax is positive (at least if leisure is non-inferior) implying that it is optimal to carry tax burden to the steady state.¹¹

6. Conclusions

In the present work we tackle the issue of taxation in presence of endogenous fertility and under critical level utilitarian preferences. From a positive standpoint we show that a rise of the tax on the family size decreases the population growth rate and increases steady state per capita consumption, and does not affect capital; on the other hand, a rise of the capital income tax reduces both steady state capital and population growth rate, and increases per capita consumption. However, the increase of the capital income tax creates temporary population and consumption bursts and reduces the steady state capital stock, while an increase in the population tax does not.

We have also analysed the effects of a fiscal policy aiming at redistributing the tax burden in such a way to maintain per capita debt unchanged. The latter tax reform implies that capital and the population growth rate move in the same direction as the change in the tax on population size, while consumption follows the direction of change of the tax on capital income. Surprisingly enough, on policy grounds the latter result suggests that an economy that wishes to increase population growth but is burdened by high public debt (such as Italy) could increase the tax on the family size *and* reduce capital income taxes correspondently, such that, in the long run, both the rate of growth of population and the capital intensity would be increased, though with the consequence of experiencing a reduction in the long run per capita consumption and a temporary reduction of the same population rate of growth.

As far as the normative analysis is concerned, we show that, at the steady state the first-best policy entails zero capital income tax and zero debt and positive taxation of the family size, no matter whether labour supply is endogenous or not. However, when only a second-best tax structure can be implemented, then nonzero tax in capital income and negative debt turn out to be optimal in case labour supply is exogenously fixed. Finally, the zero capital income tax result arises also in our model when labour supply is endogenous

¹¹ In our endogenous population economy, non-inferiority of leisure is sufficient for the labour tax to be strictly positive at the steady state. On this issue in a Chamley setting, with fixed population, see Renström (1999). For indivisible labour economies, with fixed population, normality is needed for a positive labour tax, see Basu and Renström (2007).

and taxes on labour income can be levied. The latter turn out to be positive if leisure is a non-inferior good.

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Appendix

In this Appendix we show that any allocation stemming from a competitive equilibrium satisfies implementability and feasibility constraints (18a) and (5).

By multiplying both sides of eq. (2) by $q_t e^{-\int_0^t \bar{r}(s) ds}$, integrating out the household’s budget constraint and exploiting the transversality condition we get:

$$\left(\dot{A}_t - \bar{r}_t A_t \right) q_t e^{-\int_0^t \bar{r}(s) ds} = q_t N_t e^{-\int_0^t \bar{r}(s) ds} \left(w_t - c_t - \tau_t^N \right), \text{ that is}$$

$$A_0 u'_0 = - \int_0^\infty e^{-\rho t} u'_t \left(w_t - c_t - \tau_t^N \right) N_t dt .$$

As for feasibility, write eq. 2) as $\dot{A}_t = r_t (1 - \tau_t^k) A_t + w_t N_t - c_t N - \tau_t^N N_t$. Using market clearing condition (eq. 8) we get:

$$\dot{K}_t + \dot{B}_t = r_t (1 - \tau_t^k) (K_t + B_t) + w_t N_t - c_t N - \tau_t^N N_t ;$$

moreover, by exploiting RCS and using eqs. (4a) and (4b) we get:

$$\dot{K}_t = F_t - \dot{B}_t + r_t B_t - c_t N - \tau_t^N N_t - \tau_t^k r_t (K_t + B_t) ;$$

and, finally, by exploiting debt equation (eq. 6) it descends that

$$\dot{K}_t = F_t - c_t N - g_t N_t .$$